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ON PIEZOGRAVITOCOGRAVITOELECTROMAGNETIC SHEAR-HORIZONTAL ACOUSTIC WAVES

Aleksey Anatolievich Zakharenko
International Institute of Zakharenko Waves (IIZWs)
660014, ul. Chaikovskogo, 20-304, Krasnoyarsk, Russia

ABSTRACT

This paper relates to the first centenary of the prediction of the existence of gravitational waves by Albert Einstein in 1916 and their prediction was experimentally confirmed in 2016 in one hundred years after the prediction. This work develops the theory of the wave propagation in the solids possessing the piezoelectric, piezomagnetic, and magnetoelectric effects as well as the piezogravitic, piezocogravitic, and gravitocogravitic effects, and the other exchange coeffects. Exploiting the quasi-static approximation in the theory of electromagnetism and gravitoelectromagnetism, the thermodynamics and the coupled equations of motion are developed in the common form. To simplify the problem of the wave propagation in these solids, the shear-horizontal (SH) wave propagation in the transversely isotropic materials was then treated. Considering all the aforementioned effects and coeffects, the explicit forms of the propagation velocities of the bulk and new surface acoustic waves (SH-BAW and new SH-SAW coupled with four potentials) were theoretically obtained. This theoretical work has the additional purpose to stimulate experimental measurements of all the necessary material parameters when a solid possesses all the effects and coeffects including the ones from the theory of gravitoelectromagnetism.

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INTRODUCTION

Before to start any description of very complicated problem of wave propagation in solids possessing many effects that can be taken into account, it is necessary to fist review the history and evolution of the discovery of different effects that can be revealed in the studied solids. It is necessary to state right away that the studied solids can simultaneously have the following known different effects: piezoelectric, piezomagnetic, magnetoelectric, piezogravitic, piezocogravitic, gravitocogravitic effects. These solids can also possess several coeffects that can be studied and discussed below. So, it is possible to review the problems of wave propagation from simple to more complicated.

The well-known piezoelectric effect can cause the propagation of the shear-horizontal surface acoustic waves (SH-SAWs) in the transversely isotropic (6 *mm*) piezoelectrics. This is the simplest case of SH-SAWs known as the surface Bleustein-Gulyaev (BG) wave independently discovered by Bleustein (1968) and Gulyaev (1969) in their developed theories to the end of the 1960s. There is also the second SH-SAW theoretically discovered by Bleustein (1968) for the other electrical boundary conditions. This SH-SAW is frequently called the surface Bleustein wave. However, the author of this theoretical report can use the words of the slower and faster surface BG-waves instead of the surface Bleustein-Gulyaev wave and surface Bleustein wave, respectively, to distinguish them from each other. The speeds of both the SH-SAWs must be slightly slower than the speed of the shear-horizontal bulk acoustic wave (SH-BAW) and the existence of the SH-SAWs demonstrates the fact of the instability of the SH-BAW for certain cuts and propagation directions in suitable solids. Studying the wave propagation in piezoelectrics, the well-known equations of electrostatics in the quasi-static approximation are used because the speed of light is approximately five orders faster than any acoustic wave speed. The slower and faster surface BG-waves can also propagate in piezomagnetics possessing the piezomagnetic effect when the piezoelectric and electric constants are substituted by the piezomagnetic and magnetic constants, respectively. Here the equations of magnetostatics in the quasi-static approximation are used for the same reason mentioned above.

Corresponding author e-mail: aazaaz@inbox.ru

There are also piezoelectromagnetics (magneto-electroelastics) as a class of magnetoelectric materials that can simultaneously possess both the piezoelectric and piezomagnetic effects resulting in the existence of the magnetoelectric effect. In these smart materials there is a possibility to control the magnetic subsystem by changes in the electric subsystem via the mechanical subsystem, or vice versa. Smart transversely isotropic (6 *mm*) piezoelectromagnetic (PEM) materials were only recently exhaustively treated regarding to the problems of different instabilities of the SH-BAW, i.e. the existence of various SH-SAWs when different electrical and magnetic boundary conditions are applied. Indeed, the equations of the electrostatics and magnetostatics must be also used for the problem of wave propagation in piezoelectromagnetics. There is the single review (Zakharenko, 2013a) concerning the problems of the wave propagation in piezoelectromagnetics. One decade ago Melkumyan (2007) has discovered several SH acoustic waves propagating in the transversely isotropic piezoelectromagnetics. However, only three of them can be called the Melkumyan SH-SAWs: the surface Bleustein-Gulyaev-Melkumyan (BGM) wave, the piezoelectric exchange surface Melkumyan (PEESM) wave, and the piezomagnetic exchange surface Melkumyan (PMESM) wave. The first Melkumyan PEM-SH-SAW is called the BGM wave to have an analogy with the surface BG-wave (Bleustein, 1968; Gulyaev, 1969). Following the theoretical work by Melkumyan (2007), several new PEM-SH-SAWs were also discovered in theoretical work (Zakharenko, 2010; Zakharenko, 2013b; Zakharenko, 2015a,b). It is now possible to state that more than ten new PEM-SH-SAWs can propagate in the transversely isotropic piezoelectromagnetics in contrast to two SH-SAWs existing in pure piezoelectrics or pure piezomagnetics. This is due to the fact of competition of three different effects mentioned above that can coexist in piezoelectromagnetics. The magnetoelectric effect is extremely weak effect compared with the piezoelectric or piezomagnetic effect. However, it can cause a dramatic influence on the existence of some new SH-SAWs (Zakharenko, 2015b).

It is obvious that any gravitational effect or relevant exchange coeffect can be extremely weak similar to the magnetoelectric effect. However, it is possible that consideration of some extremely weak effects can disclose the existence of some relevant new SH-SAWs that can propagate in the apt solids. Indeed, the gravitational effect can be readily recordable when very massive bodies (preferably solids) are treated. For instance, in the two-body system such as Moon-Earth, a slight but remarkable attraction of Earth surface towards Moon can be experimentally observed when Moon is orbiting Earth. Concerning the microworld when microwaves are propagating in a bulk solid or on the solid surface, it is thought that it is hard to record any changes caused by extremely small possible perturbations of local gravitational fields. Note that solids for the problem of acoustic wave propagation are naturally treated as continua but not discrete materials consisting of atoms. So, it is necessary to theoretically demonstrate that in solid continua some extremely weak gravitational effects or some relevant coeffects can cause the existence of some corresponding new SH-SAWs. This can be similar to the new SH-SAW existence caused by the extremely weak magnetoelectric effect in piezoelectromagnetics.

For the purpose of a deep study of the influence of some relevant gravitational effects on the existence of new SH-SAWs, it is natural to exploit the known equations of the gravitoelectromagnetism. These equations are similar to the well-known equations of electromagnetism (Heaviside, 1893; Maxwell, 1954; Jefimenko, 1992; Jefimenko, 2000; Jefimenko, 2006; Assis, 1994; Assis, 1999). One century ago, namely in 1916 Albert Einstein has predicted the existence of gravitational waves in the context of his theory of general relativity (Einstein, 1916). It is also known that gravitational waves propagate in a vacuum with the speed of light, namely the speed of the electromagnetic waves. So, the quasistatic approximation incorporating gravitational effects is fitting here as well. Using the equations of the gravitoelectromagnetism instead of the equations of the electromagnetism, one can find that the final explicit forms obtained for the propagating velocities in (Melkumyan, 2007; Zakharenko, 2010; Zakharenko, 2013a,b; Zakharenko, 2015a,b) can be readily rewritten down. Indeed, the utilization of the piezogravitic, piezocogravitic, and gravitocogravitic effects instead of the piezoelectric, piezomagnetic, and magnetoelectric effects, respectively, results in a substitution of piezoelectric, piezomagnetic, electric, magnetic, and magnetoelectric constants by the piezogravitic, piezocogravitic, gravitational, cogravitational, and gravitocogravitic constants, correspondingly. However, this substitution is questionable because anisotropic solids (monocrystals or composite materials) can be piezomagnetics or noncentrosymmetric piezoelectrics, or piezoelectromagnetics. This can mean that it is necessary to treat the gravitational exchange effects only in a coupe with the electromagnetic effects. Figure 1 schematically shows a solid continuum that can possess the mechanical, electrical, magnetic, gravitational (gravitoelectric), and cogravitational (gravitomagnetic) subsystems. Figure 1a shows that there is an interaction between any two subsystems somewhat directly (some exchange must occur) and via the mechanical subsystem. Figure 1b shows the simpler case when the gravitational or cogravitational subsystem can interact with the electrical or magnetic subsystem only via the mechanical subsystem.

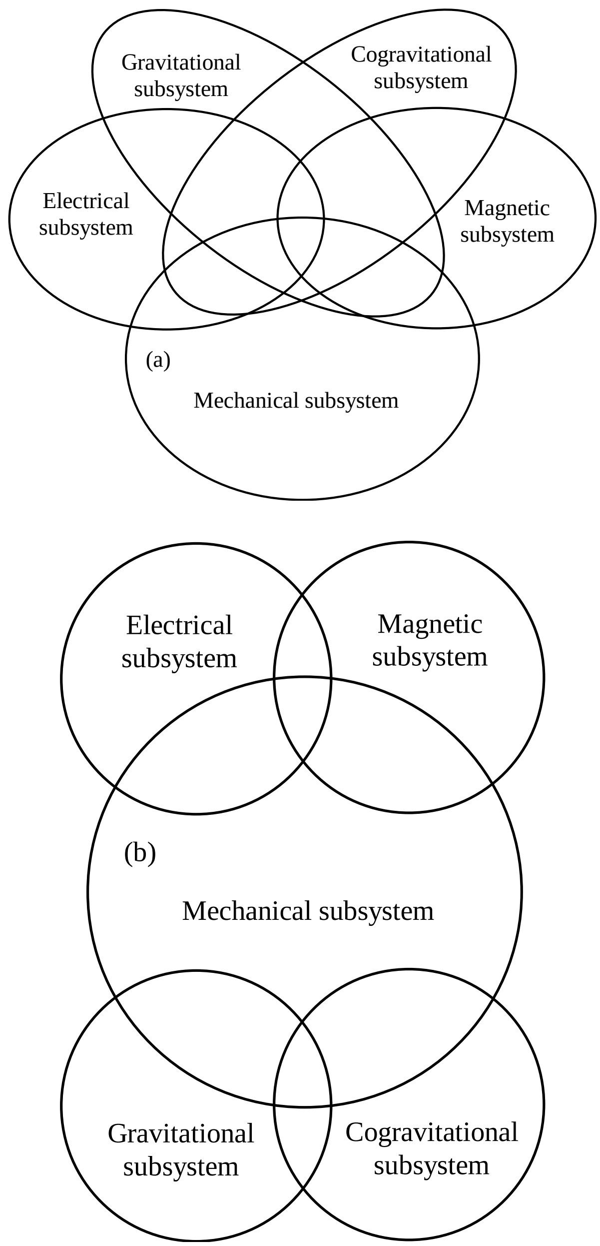

Fig. 1. The schematic demonstration of possible connections among the used subsystems such as the mechanical, electrical, magnetic, gravitational, and cogravitational ones: (a) there is an interaction between each pair of subsystem via the mechanical one and (b) the electrical subsystem can interact only with the magnetic subsystem (or vice versa) via the mechanical one and the gravitational subsystem can interact only with the cogravitational one (or vice versa) via the mechanical one.

Scientists have recently measured the gravitational equivalent of a magnetic field for the first time in a laboratory under certain special conditions. This effect is much larger than expected from general relativity. In general, scientists preferably study gravitational fields passively by observing for grasp of existing gravitational fields produced by large inertial masses such as stars or planets and there is no ability to change them, for instance, with magnetic fields. In his publication Füzfa (2016) has described one revolutionary approach for the creation of gravitational fields from well-controlled magnetic fields and observing how these magnetic fields could bend space-time. He has proposed a theoretical device based on superconducting electromagnets (modern technologies exhaustively used at CERN or the ITER reactor) for creation of detectable gravitational fields. It could disclose many new applications, for instance, in telecommunications with gravitational waves. The ability to produce, detect, and control gravitational fields would certainly be a major achievement in modern physics. So, scientific interest in the problem of interactions between the gravitational and electromagnetic waves continuously increases, for instance, see in (Hegarty, 1969; Kleidis *et al*., 2010; Forsberg *et al*., 2010). The great interest in the problem of the gravitational wave detection can be supported by the fact that the European Space Agency (ESA, the European Union) together with the National Aeronautics and Space Administration (NASA, the United States) have collaborated a series of expensive space experiments called the Laser Interferometer Space Antenna (LISA). The LISA is a proposed space-based piezoelectric device (Möhle, 2013) for gravitational waves' detection in the low frequency range from 0.1 mHz to 1.0 Hz that is not accessible by ground-based detectors. However, this expensive space journey has lost any financial support by the NASA. Also, in February, 2016, it was reported by Professor Dr. David Reitze, the executive director of the LIGO (Laser Interferometer Gravitational-Wave Observatory) that the gravitational waves were detected by the LIGO (Abbott *et al*., 2016). It is obvious that the theoretical work developed in this report does not require a multi-billion USD financial support and can be developed at an Earth laboratory, even in the International Space Station, Moon, or Mars.

The following section deals wth the thermodynamic description of a piezoelectromagnetic bulk material when the gravitational and cogravitational forces are also taken into account. The third section provides both the differential and tensor forms for the coupled equations of motion concerning the case of the shear-horizontal wave propagation. The fourth section discusses the boundary conditions that can lead to the existence of new surface SH-waves.

Thermodynamics

It is natural to consider a bulk solid continuum that simultaneously possesses the piezoelectric, piezomagnetic, and magnetoelectric effects. It is natural to assume that the gravitational (gravitoelectric) and cogravitational (gravitomagnetic) forces must be also considered. This complex continuum can be thermodynamically described by means of suitable thermodynamic variables and functions. Indeed, it is necessary to choose a thermodynamic potential to properly describe thermo gravitocogravitoelectro-magnetoelastic interactions in the continuum. It is preferable for this case to cope with the thermodynamic potential called enthalpy H_e to obtain adiabatic rather than isothermal conditions. It is well known that an adiabatic process can be characterized by the constant entropy, $S = S_0$ = const, and this thermodynamic variable illuminates a level of disorder in the system. Treating a linear case, it is possible to consider only linear terms in a Taylor series for the enthalpy H_e relative to an equilibrium condition $H_e(S_0)$. It is apparent that $S = S_0$ = const actually gives zero change, namely dS = 0. So, this thermodynamic variable can be excluded from the further analysis, for instance, see in Zakharenko (2010).

For this case, these linear terms in a Taylor series for the suitable thermodynamic potential can contain the following thermodynamic variables frequently written in the tensor forms: strain τ_{ij}, electrical field E_i, magnetic field H_i, gravitational (gravitoelectric) field GE_i, and cogravitational (gravitomagnetic) field GH_i, where the indexes i and j run from 1 to 3. Energetic terms of such complex system described by a thermodynamic potential can be naturally coupled with the following subsystems shown in Figure 1: elastic subsystem (thermodynamic variable τ_{ij}), electric subsystem (variable E_i), magnetic subsystem (variable H_i), gravitational subsystem (variable GE_i), cogravitational subsystem (variable GH_i) and thermal subsystem (entropy S).

Therefore, for the fitting thermodynamic potential T, one can write the following: $T = f(\tau_{kl}, E_k, H_k, GE_k, GH_k, S)$ and $\mathrm{d}T = f_0(\mathrm{d}\tau_{kl}, \mathrm{d}E_k, \mathrm{d}H_k, \mathrm{d}GE_k, \mathrm{d}GH_k, \mathrm{d}S = 0)$. Next, it is natural that for the problem of acoustic wave propagation in such continua, it is preferable to use the following thermodynamic functions: stress σ_{ij}, electrical displacement (induction) D_i, magnetic displacement (induction or flux) B_i, gravitational displacement (gravitoelectric induction) GD_i, and cogravitational displacement (gravitomagnetic induction) GB_i. These five thermodynamic functions depend on five independent thermodynamic variables described above as follows:

$$\sigma_{ij} = f_1(\tau_{kl}, E_k, H_k, GE_k, GH_k),$$
$$D_i = f_2(\tau_{kl}, E_k, H_k, GE_k, GH_k),$$
$$B_i = f_3(\tau_{kl}, E_k, H_k, GE_k, GH_k),$$
$$GD_i = f_4(\tau_{kl}, E_k, H_k, GE_k, GH_k),$$
$$GB_i = f_5(\tau_{kl}, E_k, H_k, GE_k, GH_k).$$

In this linear case, the coupled constitutive relations can be therefore written as follows:

$$\sigma_{ij} = C_{ijkl}\tau_{kl} - e_{kij}E_k - h_{kij}H_k - g_{kij}GE_k - f_{kij}GH_k \quad (1)$$
$$D_i = e_{ikl}\tau_{kl} + \varepsilon_{ik}E_k + \alpha_{ik}H_k + \zeta_{ik}GE_k + \xi_{ik}GH_k \quad (2)$$
$$B_i = h_{ikl}\tau_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k + \beta_{ik}GE_k + \lambda_{ik}GH_k \quad (3)$$
$$GD_i = g_{ikl}\tau_{kl} + \zeta_{ik}E_k + \beta_{ik}H_k + \gamma_{ik}GE_k + \vartheta_{ik}GH_k \quad (4)$$
$$GB_i = f_{ikl}\tau_{kl} + \xi_{ik}E_k + \lambda_{ik}H_k + \vartheta_{ik}GE_k + \eta_{ik}GH_k \quad (5)$$

In expressions from (1) to (5), the used indices i, j, k, and l run from 1 to 3. The first equation indicates that the mechanical thermodynamic function such as the stress σ_{ij} also depends on the corresponding factors at the independent thermodynamic mechanical (τ_{ij}), electrical (E_i), magnetic (H_i), gravitational (GE_i), and cogravitational (GH_i) variables. These factors represent the corresponding proportionality coefficients for the linear case and are thermodynamically defined below. They are called the elastic stiffness constants C_{ijkl}, piezoelectric constants e_{kij}, piezomagnetic coefficients h_{kij}, piezogravitic constants g_{kij}, and piezocogravitic coefficients f_{kij}.

In equations (2) and (3), the thermodynamic functions such as the electrical and magnetic displacements (D_i and B_i) also depend on the corresponding factors at the thermodynamic variables and they can be divided into two groups. The first group is for the dielectric permittivity coefficients (electrical constants) ε_{ik}, magnetic permeability coefficients (magnetic constants) μ_{ik}, and electromagnetic constants α_{ik}. The second group is for the following exchange tensors: material exchange constants ζ_{ik}, ξ_{ik}, β_{ik}, and λ_{ik} that symbolize possible exchanges between the electrical and magnetic subsystems on one side and the gravitational and cogravitational subsystems on the other side. It is necessary to keep in mind that these exchange tensors must be nonzero even in the case when their possible values are very small. However, these small material parameters must be taken into account in the common case shown in figure 1a and can be neglected for the case shown in figure 1b. It was found that consideration of very small but nonzero material parameters can be very important. This fact was demonstrated in work (Zakharenko, 2010; Zakharenko, 2013b; Zakharenko, 2015a,b) concerning the wave propagation in piezoelectromagnetics when the extremely small electromagnetic constant α can cause the existence of several new SH-SAWs. In the common case, the tensor of the electromagnetic constants α_{ik} is non symmetric in contrast to the symmetric tensors of the electrical ε_{ik} and magnetic μ_{ik} constants. However, for the cubic and transversely isotropic (6 mm) materials the tensor α_{ik} is symmetric (Schmid, 2008; Rivera, 2009). This symmetry can be also applied to other exchange tensors such as ζ_{ik}, ξ_{ik}, β_{ik}, and λ_{ik} in the first approximation, assuming that the values of the material parameters of the tensors can be very small. Similar to the exchange between the electrical and magnetic subsystems characterized by the tensor of the electromagnetic constants α_{ik} there must also exist an exchange between the gravitational and cogravitational subsystems. This exchange is taken into account by the presence of the exchange tensor ϑ_{ik} in equations (4) and (5). The tensor ϑ_{ik} can be called the tensor of the gravitocogravitic constants and it is possible to assume that the tensors α_{ik} and ϑ_{ik} can be symmetric for the same materials. Equations (4) and (5) also contain the tensors of the gravitic and cogravitic constants, γ_{ik} and η_{ik}, respectively. It is possible to require that they are symmetric similar to the tensors of ε_{ik} and μ_{ik}.

In five equations written above, the first independent thermodynamic variable such as the strain tensor τ_{ij} can be defined by the following well known relation between the strain and the mechanical displacements for small perturbations: $\tau_{ij} = 0.5(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$, where the indices i and j also run from 1 to 3. So, this relation represents the dependence of the strain tensor components τ_{ij} on the corresponding partial first derivatives of the mechanical displacement components U_1, U_2, and U_3 with respect to the real space components x_1, x_2, and x_3. Each mechanical displacement component is directed along the corresponding real space component for the rectangular coordinate system shown in Figure 2.

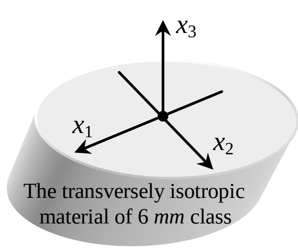

Fig. 2. The rectangular coordinate system. The coordinate beginning is situated at the vacuum-solid interface. The propagation direction is managed along the x_1-axis. The surface normal is directed along the x_3-axis. The 6-fold symmetry axis of the studied transversely isotropic (6 *mm*) material is parallel to the x_2-axis.

In equations from (1) to (5), the other independent thermodynamic variables such as the electrical field E_i, magnetic field H_i, gravitational field GE_i, and cogravitational field GH_i can be also defined by corresponding partial first derivatives. Using the corresponding potentials (electrical potential φ, magnetic potential ψ, gravitational potential Φ, and cogravitational potential Ψ) in the quasi-static (irrotational field) approximation, the components of all the fields are determined as the following partial first derivatives with respect to the real space components x_1, x_2, and x_3: $E_i = -\partial\varphi/\partial x_i$, $H_i = -\partial\psi/\partial x_i$, $GE_i = -\partial\Phi/\partial x_i$, $GH_i = -\partial\Psi/\partial x_i$. It is natural to exploit the quasi-static approximation when all the derivatives with respect to the time t in the corresponding Maxwell equations of electromagnetism (or the corresponding equations of the gravitoelectromagnetism) are omitted because the speed of the electromagnetic (or gravitational) wave is approximately five orders larger than the speed of any elastic wave (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010).

In the constitutive relations from (1) to (5), all the material tensors such as C_{ijkl}, e_{kij}, h_{kij}, g_{kij}, f_{kij}, ε_{ik}, μ_{ik}, α_{ik}, γ_{ik}, η_{ik}, ϑ_{ik}, ζ_{ik}, ξ_{ik}, β_{ik}, λ_{ik} can be thermodynamically expressed. For the thermodynamic definition of the elastic stiffness constants C_{ijkl}, these material parameters can be naturally defined from expression (1) as follows:

$$C_{ijkl} = \left(\frac{\partial\sigma_{ij}}{\partial\tau_{kl}}\right)_{E,H,GE,GH=\text{const}} \quad (6)$$

Thermodynamic definition (6) of the elastic stiffness constants C_{ijkl} states that they can be determined at constant electrical, magnetic, gravitational, and cogravitational fields. Symmetry arguments allow some simplifications of the quantity of the C_{ijkl} because the stress and strain tensors are symmetric: $\sigma_{ij} = \sigma_{ji}$ and $\tau_{ij} = \tau_{ji}$. Therefore, the stiffness tensor C_{ijkl} must also possess a corresponding degree of symmetry resulting in the following simplifications:

$$C_{ijkl} = C_{klij} = C_{jikl} = C_{klji} = C_{ijlk} = C_{lkij} = C_{jilk} = C_{lkji} \quad (7)$$

Using Voigt's notation, (3×3×3×3) tensor form (6) for the elastic stiffness constants C_{ijkl} can be rewritten in a form of (6×6) symmetric matrix (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010). The transformation procedure of a tensor form into a matrix is wellknown. For this purpose, the following rules are used for the indices: 11 → 1, 22 → 2, 33 → 3, 23 → 4, 13 → 5, 12 → 6 and therefore, $ijkl \rightarrow PQ$ and $C_{ijkl} \rightarrow C_{PQ}$, where the indices P and Q run from 1 to 6.

With equations from (1) to (5), the thermodynamic description of the piezoelectric constants e_{kij}, piezomagnetic coefficients h_{kij}, piezogravitic constants g_{kij}, and piezocogravitic coefficients f_{kij} can be correspondingly given by the following definitions:

$$e_{ijk} = -\left(\frac{\partial\sigma_{ij}}{\partial E_k}\right)_{\tau,H,GE,GH=\text{const}} = e_{ikl} = \left(\frac{\partial D_i}{\partial\tau_{kl}}\right)_{E,H,GE,GH=\text{const}} \quad (8)$$

$$h_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial H_k}\right)_{\tau,E,GE,GH=\mathrm{const}} = h_{ikl} = \left(\frac{\partial B_i}{\partial \tau_{kl}}\right)_{E,H,GE,GH=\mathrm{const}} \quad (9)$$

$$g_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial GE_k}\right)_{\tau,E,H,GH=\mathrm{const}} = g_{ikl} = \left(\frac{\partial GD_i}{\partial \tau_{kl}}\right)_{E,H,GE,GH=\mathrm{const}} \quad (10)$$

$$f_{ijk} = -\left(\frac{\partial \sigma_{ij}}{\partial GH_k}\right)_{\tau,E,H,GE=\mathrm{const}} = f_{ikl} = \left(\frac{\partial GB_i}{\partial \tau_{kl}}\right)_{E,H,GE,GH=\mathrm{const}} \quad (11)$$

It is necessary to state that the quantities of the tensors h_{kij}, e_{kij}, g_{kij}, and f_{kij} can be decreased. The symmetry arguments such as $\sigma_{ij} = \sigma_{ji}$ and $\tau_{ij} = \tau_{ji}$ can also demonstrate the corresponding degrees of symmetry for the h_{kij}, e_{kij}, g_{kij}, and f_{kij}. The symmetry influences allow the existence of the following equalities:

$$e_{kij} = e_{ijk} = e_{kji} = e_{jik} \quad (12)$$

$$h_{kij} = h_{ijk} = h_{kji} = h_{jik} \quad (13)$$

$$g_{kij} = g_{ijk} = g_{kji} = g_{jik} \quad (14)$$

$$f_{kij} = f_{ijk} = f_{kji} = f_{jik} \quad (15)$$

Exploiting Voigt's notation, all of the (3×3×3) tensor forms for the h_{kij}, e_{kij}, g_{kij}, and f_{kij} can be then rewritten as the asymmetric (6×3) or (3×6) matrices: $e_{kij} \to e_{kP}$ or $e_{ijk} \to e_{Pk}$, $h_{kij} \to h_{kP}$ or $h_{ijk} \to h_{Pk}$, $g_{kij} \to g_{kP}$ or $g_{ijk} \to g_{Pk}$, $f_{kij} \to f_{kP}$ or $f_{ijk} \to f_{Pk}$, where the index P runs from 1 to 6.

Next, the rest material tensors such as ε_{ik}, μ_{ik}, α_{ik}, γ_{ik}, η_{ik}, ϑ_{ik}, ζ_{ik}, ξ_{ik}, β_{ik}, λ_{ik} can be divided into three groups. The first group (ε_{ik}, μ_{ik}, α_{ik}) is for the electrical and magnetic subsystems and their interaction. The second (γ_{ik}, η_{ik}, ϑ_{ik}) is for the gravitational and cogravitational subsystems and their interaction. Thus, the third group contains the rest four exchange tensors. With the first group, the thermodynamic definitions for the dielectric permittivity coefficients ε_{ik}, magnetic permeability coefficients μ_{ik}, electromagnetic constants α_{ik} (see equations (2) and (3)) read:

$$\varepsilon_{ik} = \left(\frac{\partial D_i}{\partial E_k}\right)_{\tau,H,GE,GH=\mathrm{const}} \quad (16)$$

$$\mu_{ik} = \left(\frac{\partial B_i}{\partial H_k}\right)_{\tau,E,GE,GH=\mathrm{const}} \quad (17)$$

$$\alpha_{ik} = \left(\frac{\partial D_i}{\partial H_k}\right)_{\tau,E,GE,GH=\mathrm{const}} = \left(\frac{\partial B_i}{\partial E_k}\right)_{\tau,H,GE,GH=\mathrm{const}} \quad (18)$$

In the thermodynamic relations (16) and (17), the constants ε_{ik} and μ_{ik} represent symmetric tensors of the second rank (matrices): $\varepsilon_{ik} = \varepsilon_{ki}$ and $\mu_{ik} = \mu_{ki}$. It is also natural to treat $\alpha_{ik} = \alpha_{ki}$ because it is symmetric for the cubic and transversely isotropic (6 *mm*) materials (Schmid, 2008; Rivera, 2009). Indeed, the components of the tensors ε_{ik}, μ_{ik}, and α_{ik} are naturally written as (3×3) matrices (Schmid, 2008; Rivera, 2009; Zakharenko, 2010).

With expressions (4) and (5), the second group of the material tensors of the gravitic constants γ_{ik} and cogravitic constants η_{ik} can be also written as (3×3) symmetric matrices: $\gamma_{ik} = \gamma_{ki}$ and $\eta_{ik} = \eta_{ki}$. Also, it is possible also to treat the tensor of the gravitocogravitic constants ϑ_{ik} as symmetric for cubic and transversely isotropic (6 *mm*) materials. The thermodynamically defined as follows:

$$\gamma_{ik} = \left(\frac{\partial GD_i}{\partial GE_k}\right)_{\tau,E,H,GH=\mathrm{const}} \quad (19)$$

$$\eta_{ik} = \left(\frac{\partial GB_i}{\partial GH_k}\right)_{\tau,E,H,GE=\mathrm{const}} \quad (20)$$

$$\vartheta_{ik} = \left(\frac{\partial GD_i}{\partial GH_k}\right)_{\tau,E,H,GE=\mathrm{const}} = \left(\frac{\partial GB_i}{\partial GE_k}\right)_{\tau,E,H,GH=\mathrm{const}} \quad (21)$$

In the rest third group there are four exchange tensors such as ζ_{ik}, ξ_{ik}, β_{ik}, λ_{ik}. They are present in expressions from (2) to (5) of the constitutive relations and manifest possible exchange mechanisms between the electromagnetism and gravitoelectromagnetism. One has to be sure that some exchange occurs because there are recently performed experiments (Füzfa, 2016) in a laboratory on Earth concerning the evidence of creation of gravitational fields from well-controlled magnetic fields. So, it is even possible to require that the exchange tensors ζ_{ik}, ξ_{ik}, β_{ik}, and λ_{ik} must be also symmetric at least for the cubic and transversely isotropic (6 *mm*) materials. This requirement is enough for this study because it deals for simplicity with the 6 *mm* hexagonal materials. Consequently, the rest exchange tensors are thermodynamically defined as follows:

$$\zeta_{ik} = \left(\frac{\partial D_i}{\partial GE_k}\right)_{\tau,E,H,GH=\mathrm{const}} = \left(\frac{\partial GD_i}{\partial E_k}\right)_{\tau,H,GE,GH=\mathrm{const}} \quad (22)$$

$$\xi_{ik} = \left(\frac{\partial D_i}{\partial GH_k}\right)_{\tau,E,H,GE=\mathrm{const}} = \left(\frac{\partial GB_i}{\partial E_k}\right)_{\tau,H,GE,GH=\mathrm{const}} \quad (23)$$

$$\beta_{ik} = \left(\frac{\partial B_i}{\partial GE_k}\right)_{\tau,E,H,GH=\mathrm{const}} = \left(\frac{\partial GD_i}{\partial H_k}\right)_{\tau,E,GE,GH=\mathrm{const}} \quad (24)$$

$$\lambda_{ik} = \left(\frac{\partial B_i}{\partial GH_k}\right)_{\tau,E,H,GE=\mathrm{const}} = \left(\frac{\partial GB_i}{\partial H_k}\right)_{\tau,E,GE,GH=\mathrm{const}} \quad (25)$$

This study deals with wave propagation in anisotropic solid continua, i.e. crystals. This means that the propagation velocity must be different for different propagation directions in crystals. In general, all the material parameters are obtained in a crystallographic coordinate system that can naturally provide minimum set of independent material constants for simplicity. This study relates to surface acoustic SH-wave propagation coupled with all the potentials (electrical potential φ, magnetic potential ψ, gravitational potential Φ, and cogravitational potential Ψ) in such smart materials. However, surface SH-waves can be supported only in suitable propagation directions. It is necessary to rotate around the x_1-axis, x_2-axis, or x_3-axis in order to obtain a new propagation direction in the new suitable coordinate system called the work coordinate system. The new propagation direction must be directed along the x_1-axis in the work coordinate system. This situation requires a recalculation of all the values of the independent material constants. Therefore, the number of independent material constants and their values must be recalculated. It is obvious that the values of the new material constants are obtained using the values of the old ones. Exploiting the rules for tensor transformations (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010), some new values of the material constants with the indexes *i*, *j*, *k*, and *l* can be obtained by application of the transformation matrices such as a_{im}, a_{jn}, a_{kp}, and a_{lq} to the original values of the material constants with the indexes *m*, *n*, *p*, and *q*. Therefore, the transformation formulae for all the material tensors introduced above read:

$$C_{ijkl} = a_{im}a_{jn}a_{kp}a_{lq}C_{mnpq} \quad (26)$$

$$e_{ijk} = a_{im}a_{jn}a_{kp}e_{mnp} \quad (27)$$

$$h_{ijk} = a_{im}a_{jn}a_{kp}h_{mnp} \quad (28)$$

$$g_{ijk} = a_{im}a_{jn}a_{kp}g_{mnp} \quad (29)$$

$$f_{ijk} = a_{im}a_{jn}a_{kp}f_{mnp} \quad (30)$$

$$\varepsilon_{ij} = a_{im}a_{jn}\varepsilon_{mn} \quad (31)$$

$$\mu_{ij} = a_{im}a_{jn}\mu_{mn} \quad (32)$$

$$\alpha_{ij} = a_{im}a_{jn}\alpha_{mn} \quad (33)$$

$$\gamma_{ij} = a_{im}a_{jn}\gamma_{mn} \quad (34)$$

$$\eta_{ij} = a_{im}a_{jn}\eta_{mn} \quad (35)$$

$$\vartheta_{ij} = a_{im}a_{jn}\vartheta_{mn} \quad (36)$$

$$\zeta_{ij} = a_{im}a_{jn}\zeta_{mn} \quad (37)$$

$$\xi_{ij} = a_{im}a_{jn}\xi_{mn} \quad (38)$$

$$\beta_{ij} = a_{im}a_{jn}\beta_{mn} \quad (39)$$

$$\lambda_{ij} = a_{im}a_{jn}\lambda_{mn} \quad (40)$$

So, all the properly transformed material constants given by transformations from (26) to (40) will be used in the following section. The following section provides both the differential and tensor form of the coupled equations of motion. The equations of motion must be resolved for construction of apt boundary conditions' determinants for determination of propagation velocity. To obtain the propagation velocity based on the thermodynamic principles developed in this section is the main purpose of this theoretical investigation.

Coupled equations of motion

One of the common work tools in the physical acoustics is the application of the quasi-static approximation because the speed of the electromagnetic wave or gravitational wave is approximately five orders larger than the speed of any acoustic wave. Indeed, the acoustic waves propagating in solids are extremely slow in comparison with the electromagnetic (or gravitational) wave propagating in the same material. However, propagation of the acoustic waves in suitable solid continua can be naturally coupled with the electrical (φ), magnetic (ψ), gravitational (Φ), and cogravitational (Ψ) potentials in the quasi-static approximation. Using the four field equations of his electromagnetic theory, Maxwell has creatively formulated the laws of electrostatics, magnetostatics, and electromagnetism. The electrostatic and magnetostatic equilibrium equations can be written using the differential forms of the corresponding Maxwell equations which can be written as follows: $\mathrm{div}\mathbf{D} = 0$ and $\mathrm{div}\mathbf{B} = 0$. The first equality with the electrical displacement vector **D** represents Gauss's law without free charge and currents and the second equality represents a divergence of the magnetic displacement vector **B**. Using the analogy (Heaviside, 1893; Maxwell, 1954; Assis, 1994; Assis, 1999; Jefimenko, 1992; Jefimenko, 2000; Jefimenko, 2006) between the electromagnetism and gravitoelectromagnetism, it is possible to write down the gravitostatic (gravitoelectrostatic) and cogravitostatic (gravitomagnetostatic) equilibrium equations for the studied case as follows: $\mathrm{div}\mathbf{GD} = 0$ and $\mathrm{div}\mathbf{GB} = 0$, where **GD** and **GB** are the gravitational (gravitoelectrical) and cogravitational (gravitomagnetic) displacement vectors, respectively.

Further exploitation of the analogy between the electromagnetism and gravitoelectromagnetism, the governing electrostatic, magnetostatic, gravitostatic, and cogravitostatic equilibrium equations can be respectively exposed in the following differential forms: $\partial D_i/\partial x_i = 0$, $\partial B_i/\partial x_i = 0$, $\partial GD_i/\partial x_i = 0$ and $\partial GB_i/\partial x_i = 0$. These equations represent the partial first derivatives of the electrical, magnetic, gravitational, and cogravitational displacement components (i.e. D_i, B_i,

GD_i, and GB_i, respectively) with respect to the real space components x_i, where the index i runs from 1 to 3. Besides, the governing mechanical equilibrium equation is also written as the following partial first derivative of the stress tensor components σ_{ij} with respect to the components x_j (i and j run from 1 to 3): $\partial\sigma_{ij}/\partial x_j = 0$.

In the theory of the wave motions of a solid material in dependence on time t, equations of motion can be described by the following common form (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010):

$$\frac{\partial\sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 U_i}{\partial t^2} \tag{41}$$

where ρ is the mass density of the bulk solid continuum. On the right-hand side in equation (41), the partial second derivatives of the mechanical displacement components U_i with respect to time t represent corresponding accelerations with the dimension of m/s^2.

In addition to equation of motion (41), it is necessary to account the electrostatics, magnetostatics, gravitostatics, and cogravitostatics in the quasi-static approximation:

$$\frac{\partial D_i}{\partial x_j} = 0,\ \frac{\partial B_i}{\partial x_j} = 0,\ \frac{\partial GD_i}{\partial x_j} = 0,\ \frac{\partial GB_i}{\partial x_j} = 0 \tag{42}$$

It is obvious that equations (41) and (42) represent the coupled equations of motion in the differential form. The coupled equations of motion can be readily rewritten in the corresponding expended forms when the mechanical displacements U_i, electrical potential φ, magnetic potential ψ, gravitational potential Φ, and cogravitational potential Ψ are exploited. These four potentials are defined in the context above equation (6). Utilizing these four potentials for equations from (1) to (5), equations (41) and (42) take the following expanded forms:

$$\rho\frac{\partial^2 U_i}{\partial t^2} = C_{ijkl}\frac{\partial^2 U_l}{\partial x_j\partial x_k} + e_{kij}\frac{\partial^2 \varphi}{\partial x_j\partial x_k} + h_{kij}\frac{\partial^2 \psi}{\partial x_j\partial x_k} + g_{kij}\frac{\partial^2 \Phi}{\partial x_j\partial x_k} + f_{kij}\frac{\partial^2 \Psi}{\partial x_j\partial x_k} \tag{43}$$

$$0 = e_{ijk}\frac{\partial^2 U_k}{\partial x_i\partial x_j} - \varepsilon_{ij}\frac{\partial^2 \varphi}{\partial x_i\partial x_j} - \alpha_{ij}\frac{\partial^2 \psi}{\partial x_i\partial x_j} - \zeta_{ij}\frac{\partial^2 \Phi}{\partial x_i\partial x_j} - \xi_{ij}\frac{\partial^2 \Psi}{\partial x_i\partial x_j} \tag{44}$$

$$0 = h_{ijk}\frac{\partial^2 U_k}{\partial x_i\partial x_j} - \alpha_{ij}\frac{\partial^2 \varphi}{\partial x_i\partial x_j} - \mu_{ij}\frac{\partial^2 \psi}{\partial x_i\partial x_j} - \beta_{ij}\frac{\partial^2 \Phi}{\partial x_i\partial x_j} - \lambda_{ij}\frac{\partial^2 \Psi}{\partial x_i\partial x_j} \tag{45}$$

$$0 = g_{ijk}\frac{\partial^2 U_k}{\partial x_i\partial x_j} - \zeta_{ij}\frac{\partial^2 \varphi}{\partial x_i\partial x_j} - \beta_{ij}\frac{\partial^2 \psi}{\partial x_i\partial x_j} - \gamma_{ij}\frac{\partial^2 \Phi}{\partial x_i\partial x_j} - \vartheta_{ij}\frac{\partial^2 \Psi}{\partial x_i\partial x_j} \tag{46}$$

$$0 = f_{ijk}\frac{\partial^2 U_k}{\partial x_i\partial x_j} - \xi_{ij}\frac{\partial^2 \varphi}{\partial x_i\partial x_j} - \lambda_{ij}\frac{\partial^2 \psi}{\partial x_i\partial x_j} - \vartheta_{ij}\frac{\partial^2 \varphi}{\partial x_i\partial x_j} - \eta_{ij}\frac{\partial^2 \psi}{\partial x_i\partial x_j} \tag{47}$$

In equations from (43) to (47), the indexes i, j, k, and l run from 1 to 3. These five homogeneous equations represent partial differential equations of the second order. They are actually seven equations because equation (43) can be also written in the form of three equations corresponding to the mechanical displacement components U_1, U_2, and U_3. These coupled equations of motion constitute the wave propagation in a suitable solid continuum possessing the piezoelectric, piezomagnetic, piezoelectromagnetic, piezogravitic, piezocogravitic, gravitocogravitic effects and the other coeffects.

Next, it is convenient to further deal with the well known tensor form of the coupled equations of motion that can be obtained from the differential form written above. First of all, it is required to state that these homogeneous partial differential equations of the second order written above must have natural solutions in the plane wave forms (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010). Therefore, these solutions read:

$$U_I = U_I^0 \exp\left[\mathrm{j}\left(k_1x_1 + k_2x_2 + k_3x_3 - \omega t\right)\right] \tag{48}$$

where the index I runs from 1 to 7.

In solutions (48) there is the following: $U_I = U_i$ for $I = i = 1, 2, 3$; $U_4 = \varphi$, $U_5 = \psi$, $U_6 = \Phi$ and $U_7 = \Psi$. Also, U_I^0, j = $(-1)^{1/2}$, and ω stand for the initial amplitudes, imaginary unity, and angular frequency, respectively. The angular frequency ω is defined by the linear frequency ν: $\omega = 2\pi\nu$. The values of U_1^0, U_2^0, U_3^0, $U_4^0 = \varphi^0$, $U_5^0 = \psi^0$, $U_6^0 = \Phi^0$, and $U_7^0 = \Psi^0$ called the eigenvector components should be determined further. In (48), the parameters k_1, k_2, and k_3 represent the components of the wavevector **k** directed towards the wave propagation:

$(k_1, k_2, k_3) = k(n_1, n_2, n_3)$, where n_1, n_2, and n_3 are the directional cosines, namely $n_1 = 1$, $n_2 = 0$, and $n_3 \equiv n_3$. It is worth noting that the wavenumber k in the direction of wave propagation is coupled with the wavelength λ as follows: $k\lambda = 2\pi$.

It is transparent that the utilization of solutions (48) and the directional cosines for the differential form of the coupled equations from (43) to (47) can actually lead to coupled equations written in a tensor form. These homogeneous equations can be naturally written in the following compact form of the well-known Green-Christoffel equation (Zakharenko, 2010):

$$\left(GL_{IJ} - \delta_{IJ}\rho V_{ph}\right)U_I^0 = 0 \qquad (49)$$

where the indices I and J run from 1 to 7 and the phase velocity is defined by $V_{ph} = \omega/k$.

In equation (49), GL_{IJ} stands for the components of the modified tensor in the well-known Green-Christoffel equation (Zakharenko, 2010). δ_{IJ} represents the Kronecker delta-function with the following conditions: $\delta_{IJ} = 1$ for $I = J < 4$, $\delta_{IJ} = 0$ for $I \neq J$, and $\delta_{44} = \delta_{55} = \delta_{66} = \delta_{77} = 0$. It is also fundamental to state that the symmetric modified Green-Christoffel tensor GL_{IJ}, i.e. $GL_{IJ} = GL_{JI}$, can have only 28 independent tensor components. Compact form (49) represents the common problem for determination of the eigenvalues and eigenvectors. In this case, the suitable values of n_3 for the corresponding phase velocity represent the eigenvalues and a corresponding eigenvector $\left(U_1^0, U_2^0, U_3^0, U_4^0 = \varphi^0, U_5^0 = \psi^0, U_6^0 = \Phi^0, U_7^0 = \Psi^0\right)$ should exist for each of the suitable eigenvalues.

However, this report relates to the study of the SH-wave propagation and therefore, only suitable several equations must be used from common compact form (49) corresponding to fitting propagation directions. According to excellent books (Dieulesaint and Royer, 1980; Auld, 1990), it is possible to find high symmetry propagation directions in crystals relating to all classes of symmetry, but the lowest triclinic symmetry. In such propagation directions, tensor form (49) can consist of two independent sets of homogeneous equations due to the fact that some GL-tensor components can become equal to zero when acoustic waves propagate in certain directions on certain cuts. In some certain directions (Dieulesaint and Royer, 1980; Auld, 1990) of wave propagation, the in-plane polarized waves can be coupled with the four potentials (electrical φ, magnetic ψ, gravitational Φ, and cogravitational Ψ potentials) and the anti-plane polarized (SH) waves represent purely mechanical waves. Therefore, the corresponding eigenvectors are respectively written as follows: $\left(U_1^0, U_3^0, U_4^0, U_5^0, U_6^0, U_7^0\right)$ and $\left(U_2^0\right)$. In the other certain directions (Dieulesaint and Royer, 1980; Auld, 1990), the in-plane polarized waves represent purely mechanical waves and the anti-plane polarized (SH) waves can be coupled with the four potentials. This case corresponds to the following eigenvectors: $\left(U_1^0, U_3^0\right)$ and $\left(U_2^0, U_4^0, U_5^0, U_6^0, U_7^0\right)$.

This study has an interest in investigation of the pure SH-waves in the suitable high symmetry propagation directions in the transversely isotropic materials of class 6 *mm*. There are certain cuts and certain propagation directions in such materials (Dieulesaint and Royer, 1980; Auld, 1990; Gulyaev, 1998; Zakharenko, 2010) in which the propagation of the pure SH-waves can be coupled with the four potentials. Figure 2 shows the suitable propagation direction managed along the x_1-axis in the work coordinate system (x_1, x_2, x_3) in which the six fold symmetry axis is directed along the x_2-axis. The work coordinate system was obtained from the original crystallographic coordinate system (x'_1, x'_2, x'_3) in which the six fold symmetry axis is directed along the surface normal. In this case, the SH-wave has the mechanical displacement component U_2 directed along the x_2-axis. In the studied propagation direction, it is unnecessary to expand compact tensor form (49) because it actually decomposes into two independent parts and there is only an interest in the part corresponding to the pure SH-wave propagation.

Dealing only with the suitable GL-tensor components of compact tensor form (49) representing the coupled equations of motion, the SH-wave propagation coupled with the four potentials can be then expressed by the following five homogeneous equations:

$$\begin{pmatrix} GL_{22} - \rho V_{ph}^2 & GL_{24} & GL_{25} & GL_{26} & GL_{27} \\ GL_{42} & GL_{44} & GL_{45} & GL_{46} & GL_{47} \\ GL_{52} & GL_{54} & GL_{55} & GL_{56} & GL_{57} \\ GL_{62} & GL_{64} & GL_{65} & GL_{66} & GL_{67} \\ GL_{72} & GL_{74} & GL_{75} & GL_{76} & GL_{77} \end{pmatrix} \begin{pmatrix} U_2^0 \\ U_4^0 \\ U_5^0 \\ U_6^0 \\ U_7^0 \end{pmatrix} =$$

$$\begin{pmatrix} C\left[m - \left(V_{ph}/V_{t4}\right)^2\right] & em & hm & gm & fm \\ em & -\varepsilon m & -\alpha m & -\zeta m & -\xi m \\ hm & -\alpha m & -\mu m & -\beta m & -\lambda m \\ gm & -\zeta m & -\beta m & -\gamma m & -\vartheta m \\ fm & -\xi m & -\lambda m & -\vartheta m & -\eta m \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \\ \Phi^0 \\ \Psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (50)$$

where $m = 1 + n_3^2$ and $\left(U^0, \varphi^0, \psi^0, \Phi^0, \Psi^0\right) = \left(U_2^0, U_4^0, U_5^0, U_6^0, U_7^0\right)$.

In equation (50), the independent material constants for the case are $C = C_{44} = C_{66}$, $e = e_{16} = e_{34}$, $h = h_{16} = h_{34}$, $g = g_{16} = g_{34}$, $f = f_{16} = f_{34}$, $\varepsilon = \varepsilon_{11} = \varepsilon_{33}$, $\mu = \mu_{11} = \mu_{33}$, $\alpha = \alpha_{11} = \alpha_{33}$, $\gamma = \gamma_{11} = \gamma_{33}$, $\eta = \eta_{11} = \eta_{33}$, $\vartheta = \vartheta_{11} = \vartheta_{33}$, $\zeta = \zeta_{11} = \zeta_{33}$, $\xi = \xi_{11} = \xi_{33}$, $\beta = \beta_{11} = \beta_{33}$, $\lambda = \lambda_{11} = \lambda_{33}$. The suitable eigenvalues $n_3 = k_3/k$ can be found when the determinant of the coefficient matrix in equations (50) equals to zero. Therefore, it is possible to write down this determinant of the coefficient matrix already in the following convenient form consisting of five cofactors:

$$\begin{vmatrix} C\left[m-\left(V_{ph}/V_{t4}\right)^2\right] & e & h & g & f \\ em & -\varepsilon & -\alpha & -\zeta & -\xi \\ hm & -\alpha & -\mu & -\beta & -\lambda \\ gm & -\zeta & -\beta & -\gamma & -\vartheta \\ fm & -\xi & -\lambda & -\vartheta & -\eta \end{vmatrix} \times m \times m \times m \times m = 0 \quad (51)$$

The first factor representing the determinant in equation (51) is quite complicated and the rest ones give the following four pairs of identical eigenvalues:

$$n_3^{(1,2)} = n_3^{(3,4)} = n_3^{(5,6)} = n_3^{(7,8)} = \pm \mathrm{j} \quad (52)$$

Expanding the determinant in equation (51) leads to the following fifth pair of the eigenvalues:

$$n_3^{(9,10)} = \pm \mathrm{j}\sqrt{1-\left(V_{ph}/V_{temgc}\right)^2} \quad (53)$$

where

$$V_{temgc} = \sqrt{C/\rho}\left(1+K_{emgc}^2\right)^{1/2} \quad (54)$$

$$K_{emgc}^2 = \frac{A_1}{A_2} \quad (55)$$

$$\begin{aligned} A_1 &= e^2\left(\mu\gamma\eta + 2\beta\lambda\vartheta - \lambda^2\gamma - \beta^2\eta - \vartheta^2\mu\right) \\ &+ h^2\left(\varepsilon\gamma\eta + 2\zeta\xi\vartheta - \vartheta^2\varepsilon - \zeta^2\eta - \xi^2\gamma\right) \\ &+ g^2\left(\varepsilon\mu\eta + 2\alpha\xi\lambda - \lambda^2\varepsilon - \alpha^2\eta - \xi^2\mu\right) \\ &+ f^2\left(\varepsilon\mu\gamma + 2\alpha\beta\zeta - \beta^2\varepsilon - \alpha^2\gamma - \zeta^2\mu\right) \\ &+ 2eh\left(\vartheta^2\alpha + \zeta\beta\eta + \xi\gamma\lambda - \alpha\gamma\eta - \zeta\lambda\vartheta - \xi\beta\vartheta\right) \\ &+ 2eg\left(\alpha\beta\eta + \lambda^2\zeta + \xi\vartheta\mu - \alpha\lambda\vartheta - \zeta\mu\eta - \xi\beta\lambda\right) \\ &+ 2ef\left(\alpha\gamma\lambda + \zeta\vartheta\mu + \beta^2\xi - \alpha\beta\vartheta - \zeta\beta\lambda - \xi\mu\gamma\right) \\ &+ 2hg\left(\varepsilon\lambda\vartheta + \zeta\alpha\eta + \xi^2\beta - \varepsilon\eta\beta - \zeta\lambda\xi - \xi\vartheta\alpha\right) \\ &+ 2hf\left(\varepsilon\beta\vartheta + \zeta^2\lambda + \xi\alpha\gamma - \varepsilon\lambda\gamma - \zeta\vartheta\alpha - \xi\zeta\beta\right) \\ &+ 2gf\left(\varepsilon\beta\lambda + \alpha^2\vartheta + \xi\mu\zeta - \varepsilon\mu\vartheta - \alpha\zeta\lambda - \alpha\beta\xi\right) \end{aligned} \quad (56)$$

$$\begin{aligned} A_2 &= C\left(\varepsilon\mu - \alpha^2\right)\left(\gamma\eta - \vartheta^2\right) \\ &+ C\left(\beta^2\xi^2 - \xi^2\mu\gamma - \beta^2\varepsilon\eta\right) + C\left(\lambda^2\zeta^2 - \lambda^2\varepsilon\gamma - \zeta^2\mu\eta\right) \\ &+ 2C\left(\gamma\alpha\xi\lambda + \eta\alpha\beta\zeta + \varepsilon\beta\lambda\vartheta + \mu\zeta\xi\vartheta - \zeta\xi\beta\lambda - \alpha\zeta\lambda\vartheta - \alpha\beta\xi\vartheta\right) \end{aligned} \quad (57)$$

Expressions (54) and (55) represent the definitions for the four-potential shear-horizontal bulk acoustic wave (4P-SH-BAW) and the coefficient of the electromagnetogravitocogravitomechanical coupling (CEMGCMC), respectively. This coefficient can be dramatically reduced for the case of $\zeta = 0$, $\xi = 0$, $\beta = 0$, $\lambda = 0$ when there is no direct exchange between the electrical (magnetic) subsystem and gravitational (cogravitational) subsystem. The reduced coefficient reads:

$$K_{emgc}^{*2} = K_{em}^2 + K_{gc}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha eh}{C\left(\varepsilon\mu - \alpha^2\right)} + \frac{\eta g^2 + \gamma f^2 - 2\vartheta gf}{C\left(\gamma\eta - \vartheta^2\right)} \quad (58)$$

where

$$K_{em}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha eh}{C\left(\varepsilon\mu - \alpha^2\right)} = \frac{e\left(e\mu - h\alpha\right) - h\left(e\alpha - h\varepsilon\right)}{C\left(\varepsilon\mu - \alpha^2\right)} = \frac{eM_2 - hM_1}{CM_3} \quad (59)$$

$$K_{gc}^2 = \frac{\eta g^2 + \gamma f^2 - 2\vartheta gf}{C\left(\gamma\eta - \vartheta^2\right)} = \frac{g\left(g\eta - f\vartheta\right) - f\left(g\vartheta - f\gamma\right)}{C\left(\gamma\eta - \vartheta^2\right)} = \frac{gM_5 - fM_4}{CM_6} \quad (60)$$

Definitions (59) and (60) stand for the coefficient of the magnetoelectromechanical coupling (CMEMC) and the coefficient of the gravitocogravitomechanical coupling (CGCMC), respectively. They depend on the following corresponding coupling mechanisms:

$$M_1 = e\alpha - h\varepsilon \quad (61)$$

$$M_2 = e\mu - h\alpha \quad (62)$$

$$M_3 = \varepsilon\mu - \alpha^2 \quad (63)$$

$$M_4 = g\vartheta - f\gamma \quad (64)$$

$$M_5 = g\eta - f\vartheta \quad (65)$$

$$M_6 = \gamma\eta - \vartheta^2 \quad (66)$$

The coupling mechanisms M_1, M_2, and M_3 are discussed in (Zakharenko, 2013c) and the others are introduced in this study. Using reduced coefficient (58), the reduced 4P-SH-BAW speed can be inscribed as follows:

$$V_{temgc}^{*}=\sqrt{C/\rho}\left(1+K_{emgc}^{*2}\right)^{1/2} \quad (67)$$

With known eigenvalues (52) and (53), it is now possible to find all the suitable eigenvectors. This is a quite complicated problem. Therefore, the appendix below lists all the suitable eigenvectors. The reader can find in the appendix that there are several suitable cases. Using the eigenvalues and the corresponding eigenvectors, it is possible to write down the complete parameters based on expression (48) and to exploit them in the apt boundary conditions. This is the purpose of the following section.

Boundary conditions leading to new SH-SAW

First of all, based on definition (48), it is indispensable to write down the explicit forms for the following complete parameters in the plane wave forms: the complete mechanical displacement $U_2^{\Sigma}=U^{\Sigma}$, complete electrical potential $U_4^{\Sigma}=\varphi^{\Sigma}$, complete magnetic potential $U_5^{\Sigma}=\psi^{\Sigma}$, complete gravitational potential $U_6^{\Sigma}=\Phi^{\Sigma}$, and complete cogravitational potential $U_7^{\Sigma}=\Psi^{\Sigma}$. These complete parameters are very important and used further to construct the determinants of the boundary conditions. These complete parameters can be naturally introduced in the following forms:

$$U_2^{\Sigma}=U^{\Sigma}=\sum_{s=1,3,5,7,9}F^{(s)}U_2^{0(s)}\exp\left[\mathrm{j}\left(k_1x_1+k_2x_2+k_3^{(s)}x_3-\omega t\right)\right]$$
$$=FU^{0(1)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(1)}x_3-V_{ph}t\right)\right]+F_9U^{0(9)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(9)}x_3-V_{ph}t\right)\right] \quad (68)$$

$$U_4^{\Sigma}=\varphi^{\Sigma}=\sum_{s=1,3,5,7,9}F^{(s)}U_4^{0(s)}\exp\left[\mathrm{j}\left(k_1x_1+k_2x_2+k_3^{(s)}x_3-\omega t\right)\right]$$
$$=F\varphi^{0(1)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(1)}x_3-V_{ph}t\right)\right]+F_9\varphi^{0(9)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(9)}x_3-V_{ph}t\right)\right] \quad (69)$$

$$U_5^{\Sigma}=\psi^{\Sigma}=\sum_{s=1,3,5,7,9}F^{(s)}U_5^{0(s)}\exp\left[\mathrm{j}\left(k_1x_1+k_2x_2+k_3^{(s)}x_3-\omega t\right)\right]$$
$$=F\psi^{0(1)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(1)}x_3-V_{ph}t\right)\right]+F_9\psi^{0(9)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(9)}x_3-V_{ph}t\right)\right] \quad (70)$$

$$U_6^{\Sigma}=\Phi^{\Sigma}=\sum_{s=1,3,5,7,9}F^{(s)}U_6^{0(s)}\exp\left[\mathrm{j}\left(k_1x_1+k_2x_2+k_3^{(s)}x_3-\omega t\right)\right]$$
$$=F\Phi^{0(1)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(1)}x_3-V_{ph}t\right)\right]+F_9\Phi^{0(9)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(9)}x_3-V_{ph}t\right)\right] \quad (71)$$

$$U_7^{\Sigma}=\Psi^{\Sigma}=\sum_{s=1,3,5,7,9}F^{(s)}U_7^{0(s)}\exp\left[\mathrm{j}\left(k_1x_1+k_2x_2+k_3^{(s)}x_3-\omega t\right)\right]$$
$$=F\Psi^{0(1)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(1)}x_3-V_{ph}t\right)\right]+F_9\Psi^{0(9)}\exp\left[\mathrm{j}k\left(x_1+n_3^{(9)}x_3-V_{ph}t\right)\right] \quad (72)$$

where $F = F^{(1)} + F^{(3)} + F^{(5)} + F^{(7)}$ and $F_9 = F^{(9)}$.

It is clearly seen in the complete parameters written above that one deals here with a five-partial wave because each complete parameter must be formed by five terms due to the summation over the index s = 1, 3, 5, 7, 9. This summation corresponds to five suitable eigenvalues of ten defined by expressions (52) and (53). The suitable eigenvalues n_3 are those with a negative sign ($x_3 < 0$ in the solid shown in Figure 2. In order to have the wave damping towards the depth of the solid because this report has an interest in a study of surface wave propagation localized at the interface between two different continua, namely a vacuum and the solid. This is usual thing for investigation of surface wave propagation in solids (Dieulesaint and Royer, 1980; Auld, 1990; Zakharenko, 2010). So, for these five-partial waves there are the following five weight factors $F^{(1)}$, $F^{(3)}$, $F^{(5)}$, $F^{(7)}$, and $F^{(9)}$. The complete parameters depend on them. However, it is clearly seen in expression (52) that there are four identical eigenvalues n_3 and they will give the same eigenvectors. As a result, all the complete parameters can be written down in convenient and simplified forms with only two weight factors such as F and F_9 defined right away after expression (72). With F and F_9, it is possible to conclude that these five-partial waves can be introduced as some hidden two-partial waves. This fact can be used further for determination of the propagation velocity of the acoustic waves coupled with the four potentials: four-potential shear-horizontal surface acoustic wave or 4P-SH-SAW.

The boundary conditions used in this theoretical report relates to the interface between a vacuum and the solid. The mechanical boundary condition for the mechanical subsystem is the mechanically free surface of the solid, i.e. the normal component of the stress tensor σ_{32} must vanish at the interface between the solid surface and a vacuum: $\sigma_{32}(x_3 = 0) = 0$. Using expression (1), this condition reads:

$$\sigma_{32}=\sum_{s=1,3,5,7,9}F^{(s)}\left[Ck_3^{(s)}U^{0(s)}+ek_3^{(s)}\varphi^{0(s)}+hk_3^{(s)}\psi^{0(s)}+gk_3^{(s)}\Phi^{0(s)}+fk_3^{(s)}\Psi^{0(s)}\right]=0$$

The electrical boundary condition for the electrical subsystem is that the electrical potential must vanish at x_3 = 0, i.e. $\varphi=\sum_{s=1,3,5,7,9}F^{(s)}\varphi^{0(s)}=0$ representing the electrically closed case (Al'shits *et al*., 1992). The magnetic boundary condition at x_3 = 0 for the magnetic subsystem is as follows: $\psi=\sum_{s=1,3,5,7,9}F^{(s)}\psi^{0(s)}=0$ representing the magnetically open case (Al'shits *et al*., 1992). Analogically, for the gravitational subsystem and the cogravitational subsystem it is possible to require that both the gravitational and cogravitational potentials must vanish at the interface x_3 = 0: $\Phi=\sum_{s=1,3,5,7,9}F^{(s)}\Phi^{0(s)}=0$

and $\Psi = \sum_{s=1,3,5,7,9} F^{(s)}\Psi^{0(s)} = 0$. It is thought that these boundary conditions are the most simple and more complicated boundary conditions will be not treated in this report.

Therefore, these five boundary conditions lead to five homogeneous equations written the following matrix form:

$$\begin{pmatrix} e\varphi^{0(1)}+h\psi^{0(1)} + g\Phi^{0(1)}+f\Psi^{0(1)} & e\varphi^{0(3)}+h\psi^{0(3)} + g\Phi^{0(3)}+f\Psi^{0(3)} & e\varphi^{0(5)}+h\psi^{0(5)} + g\Phi^{0(5)}+f\Psi^{0(5)} & e\varphi^{0(7)}+h\psi^{0(7)} + g\Phi^{0(7)}+f\Psi^{0(7)} & b(CU^{0(9)} + e\varphi^{0(9)}+h\psi^{0(9)} + g\Phi^{0(9)}+f\Psi^{0(9)}) \\ \varphi^{0(1)} & \varphi^{0(3)} & \varphi^{0(5)} & \varphi^{0(7)} & \varphi^{0(9)} \\ \psi^{0(1)} & \psi^{0(3)} & \psi^{0(5)} & \psi^{0(7)} & \psi^{0(9)} \\ \Phi^{0(1)} & \Phi^{0(3)} & \Phi^{0(5)} & \Phi^{0(7)} & \Phi^{0(9)} \\ \Psi^{0(1)} & \Psi^{0(3)} & \Psi^{0(5)} & \Psi^{0(7)} & \Psi^{0(9)} \end{pmatrix} \times \begin{pmatrix} F^{(1)} \\ F^{(3)} \\ F^{(5)} \\ F^{(7)} \\ F^{(9)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (73)$$

where $b = \sqrt{1-\left(V_{ph}/V_{temgc}\right)^2}$.

It is well-known that a set of homogeneous equations has a solution when the determinant of the coefficient matrix is equal to zero. The experienced reader can find that the determinant of the coefficient matrix in expression (73) is always equal to zero because the first, second, third, and fourth columns of the determinant are identical due to four identical eigenvalues (52). It is obvious that the identical eigenvalues give identical eigenvector components that can be found in the appendix, for instance, see in definitions (A18) and (A19) or (A30) and (A31), or the others. Indeed, it is well-known fact that a matrix determinant is equal to zero when there are two (several) identical columns or two (several) identical rows. This is also true when a column represents a linear combination of two (several) columns and or a row represents a linear combination of two (several) rows. However, this fact that there are identical columns in expression (73) does not determine the propagation velocity because all the eigenvector components do not depend on the propagation velocity, i.e. the phase velocity V_{ph} that must be found. The main peculiarity of the studied case is that only the factor b defined right away after equation (73) depends on the V_{ph}. The factor b is only present in the first row and the last column of the matrix determinant. For the sound determination of the propagation velocity, one has to treat the rows instead of the columns of the matrix determinant in (73). It is blatant that the first row actually represents a linear combination of all the rest rows. Indeed, the reader can successively subtract the second, third, fourth, and fifth rows with the factors of e, h, g, and f, respectively, from the first row and the certain value for the propagation velocity can be soundly obtained.

For this purpose, it is convenient to deal with an equivalent set of two instead of five homogeneous equations. With the five homogeneous equations written in matrix form (73) and the weight factors F and F_9 defined after expression (72), it is possible to introduce the following equivalent set of two homogeneous equations for the determination of the propagation velocity:

$$\begin{pmatrix} e\varphi^{0(1)}+h\psi^{0(1)}+g\Phi^{0(1)}+f\Psi^{0(1)} & b\left(CU^{0(9)}+e\varphi^{0(9)}+h\psi^{0(9)}+g\Phi^{0(9)}+f\Psi^{0(9)}\right) \\ e\varphi^{0(1)}+h\psi^{0(1)}+g\Phi^{0(1)}+f\Psi^{0(1)} & e\varphi^{0(9)}+h\psi^{0(9)}+g\Phi^{0(9)}+f\Psi^{0(9)} \end{pmatrix} \times \begin{pmatrix} F \\ F_9 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (74)$$

It is clearly seen in reduced set (74) that the first row in reduced set (74) represents the first row on complete set (73) and the second row in set (74) represents a linear combination of the rest rows in set (73). It is obvious that in reduced set (74), it is unnecessary to use the second, third, and fourth columns from complete set (73) because they are identical to the first column. This is the usual procedure to reduce a complicated set of equations by replacing it with a more simplified but equivalent set of equations. This can be convenient when more complicated case can be treated in the future. Exploiting reduced set (74), the propagation velocity can be determined from the following common form:

$$b = \sqrt{1-\left(V_{ph}/V_{temgc}\right)^2} = \frac{e\varphi^{0(9)}+h\psi^{0(9)}+g\Phi^{0(9)}+f\Psi^{0(9)}}{CU^{0(9)}+e\varphi^{0(9)}+h\psi^{0(9)}+g\Phi^{0(9)}+f\Psi^{0(9)}} \quad (75)$$

All the eigenvector components such as $U^{0(9)}$, $\varphi^{0(9)}$, $\psi^{0(9)}$, $\Phi^{0(9)}$, and $\Psi^{0(9)}$ can be found in the appendix that offers six different cases, each of which contains two pairs of different eigenvectors. Also, the weight factors F and F_9 can be determined from the first equation in set (74). They can be exposed as follows:

$$F = -b\left(CU^{0(9)}+e\varphi^{0(9)}+h\psi^{0(9)}+g\Phi^{0(9)}+f\Psi^{0(9)}\right) \quad (76)$$

$$F_9 = e\varphi^{0(1)}+h\psi^{0(1)}+g\Phi^{0(1)}+f\Psi^{0(1)} \quad (77)$$

Expression (75) can be also obtained from complete set (73) by a successive subtraction of the second, third, fourth, and fifth rows with the factors of e, h, g, and f, respectively, from the first row. This was already mentioned in the context above expression (74). The reader can check that the utilization of any of possible eigenvectors given in the appendix by formulae (A19) and (A31) soundly leads to the following propagation velocity of the new 4P-SH-SAW:

$$V_{newSAW} = V_{temgc}\left[1-\left(\frac{K_{emgc}^2}{1+K_{emgc}^2}\right)^2\right]^{1/2} \quad (78)$$

where the 4P-SH-BAW velocity V_{temgc} and the coefficient of the electromagnetogravitocogravito mechanical coupling (CEMGCMC) K_{emgc}^2 are respectively defined by formulae (54) and (55).

Also, the CEMGCMC K_{emgc}^2 can be dramatically reduced for the case of ζ = 0, ξ = 0, β = 0, λ = 0. This is the case of no direct exchange between the electrical (magnetic) subsystem and gravitational (cogravitational) subsystem. For this case, the reduced CEMGCMC K_{emgc}^{*2} defined by (58) must be used along with the other 4P-SH-BAW velocity V_{temgc}^* defined by formula (67). Therefore, the value of the new 4P-SH-SAW velocity can be calculated with the following formula:

$$V_{newSAW}^* = V_{temgc}^*\left[1-\left(\frac{K_{emgc}^{*2}}{1+K_{emgc}^{*2}}\right)^2\right]^{1/2} = V_{temgc}^*\left[1-\left(\frac{K_{em}^2+K_{gc}^2}{1+K_{em}^2+K_{gc}^2}\right)^2\right]^{1/2} \quad (79)$$

where the coefficient of the magnetoelectromechanical coupling (CMEMC) K_{em}^2 and the coefficient of the gravitocogravito mechanical coupling (CGCMC) K_{gc}^2 are respectively defined by (59) and (60).

It is also necessary to discuss the case when the gravitational and cogravitational effects can be neglected, i.e. the material parameters g = 0 and f = 0 resulting in $K_{gc}^2 = 0$. For this case, reduced velocity (84) further reduces to the velocity V_{BGM} of the surface Bleustein-Gulyaev-Melkumyan (BGM) wave (Melkumyan, 2007; Zakharenko, 2010, 2013a) discovered by Melkumyan (2007). This velocity reads:

$$V_{BGM} = V_{tem}\left[1-\left(\frac{K_{em}^2}{1+K_{em}^2}\right)^2\right]^{1/2} \quad (80)$$

where $V_{tem} = \sqrt{C/\rho}\left(1+K_{em}^2\right)^{1/2}$ stands for the SH-BAW velocity coupled with both the electrical and magnetic potentials.

The surface BGM wave can propagate in the piezoelectromagnetic smart materials, in which more than ten SH-SAWs were recently discovered pertaining to different boundary conditions. This report has no purpose to treat the other boundary conditions different from those used in this section. Indeed, the four-potential wave propagation is significantly more complicated case compared with the wave propagation in piezoelectromagnetic materials. Also, one can find in formula (80) that a substitution of K_{gc}^2 instead of K_{em}^2 can result in the existence of new piezogravitoco gravitational wave because neither Bleustein nor Gulyaev, nor Melkumyan has studied the gravitational effects. However, none has reported that such wave can be recorded at the current level of experimental development. Maybe this is a problem for this (next) century.

The connection between the surface BGM wave and the well-known surface Bleustein-Gulyaev (BG) wave can be also discussed. Indeed, h = 0 results in $K_{em}^2 \to K_e^2 = e^2/C\varepsilon$ and $V_{tem} \to V_{te} = \sqrt{C/\rho}\left(1+K_e^2\right)^{1/2}$ in formula (80). The coefficient of the electromechanical coupling K_e^2 and the velocity V_{te} of the SH-BAW coupled with the electrical potential φ are the characteristics for a pure piezoelectrics. On the other hand, e = 0 results in $K_{em}^2 \to K_m^2 = h^2/C\mu$ and $V_{tem} \to V_{tm} = \sqrt{C/\rho}\left(1+K_m^2\right)^{1/2}$ in formula (80). This is the case of the wave propagation in a pure piezomagnetics characterized by the coefficient of the magneto mechanical coupling K_m^2 and the velocity V_{tm} of the SH-BAW coupled with the magnetic potential ψ. Therefore, the velocity of the surface BG-wave propagating in a pure piezoelectrics or pure piezomagnetics can be calculated with the following well-known formulae (Bleustein, 1968; Gulyaev, 1969):

$$V_{BG} = V_{te}\left[1-\left(\frac{K_e^2}{1+K_e^2}\right)^2\right]^{1/2} \quad (81)$$

$$V_{BG} = V_{tm}\left[1-\left(\frac{K_m^2}{1+K_m^2}\right)^2\right]^{1/2} \quad (82)$$

For the case when some suitable materials can possess the mechanical, magnetic, and gravitational subsystems, formula (80) must be replaced by formula (83) written below. This can be the case analogical to the experimentally realized one by Professor André Füzfa (2016) when the magnetic and gravitational forces can interact, i.e. the gravitational field can be controlled by the magnetic field. If the magnetic subsystem can interact with the cogravitational subsystem, the final expression for the new propagation velocity is given by formula (84) written below.

$$V_{1new} = V_{tmg}\left[1-\left(\frac{K_{mg}^2}{1+K_{mg}^2}\right)^2\right]^{1/2} \quad (83)$$

$$V_{2new} = V_{tmc}\left[1-\left(\frac{K_{mc}^2}{1+K_{mc}^2}\right)^2\right]^{1/2} \quad (84)$$

where $V_{tmg} = \sqrt{C/\rho}\left(1+K_{mg}^2\right)^{1/2}$ stands for the velocity of the SH-BAW coupled with both the magnetic and gravitational potentials and $V_{tmc} = \sqrt{C/\rho}\left(1+K_{mc}^2\right)^{1/2}$ stands for the velocity of the SH-BAW coupled with both the magnetic and cogravitational potentials.

In formula (83), the introduced material parameter K_{mg}^2 can be called the coefficient of magnetogravito mechanical coupling (CMGMC). In expression (84), the introduced material parameter K_{mc}^2 can be analogically called the coefficient of magnetocogravito mechanical coupling (CMCMC). They are respectively defined by

$$K_{mg}^2 = \frac{\mu g^2 + \gamma h^2 - 2\beta hg}{C\left(\mu\gamma - \beta^2\right)} = \frac{g\left(g\mu - h\beta\right) - h\left(g\beta - h\gamma\right)}{C\left(\mu\gamma - \beta^2\right)} = \frac{gM_8 - hM_7}{CM_9} \quad (85)$$

$$K_{gc}^2 = \frac{\mu f^2 + \eta h^2 - 2\lambda hf}{C\left(\mu\eta - \lambda^2\right)} = \frac{f\left(f\mu - h\lambda\right) - h\left(f\lambda - h\eta\right)}{C\left(\mu\eta - \lambda^2\right)} = \frac{fM_{11} - hM_{10}}{CM_{12}} \quad (86)$$

In definitions (85) and (86), the following corresponding mechanisms of coupling are introduced:

$$M_7 = g\beta - h\gamma \quad (87)$$

$$M_8 = g\mu - h\beta \quad (88)$$

$$M_9 = \mu\gamma - \beta^2 \quad (89)$$

$$M_{10} = f\lambda - h\eta \quad (90)$$

$$M_{11} = f\mu - h\lambda \quad (91)$$

$$M_{12} = \mu\eta - \lambda^2 \quad (92)$$

CONCLUSION

This theoretical report has predicted the existence of the new four-potential shear-horizontal surface acoustic wave (4P-SH-SAW) propagation in suitable solids when the wave motion is coupled with the following four potentials: the electrical potential φ, magnetic potential ψ, gravitational potential Φ, and cogravitational potential Ψ. The velocity of the new 4P-SH-SAW was obtained in an explicit form. The obtained theoretical results can be used for further development of some problems of gravitation, the problem of 4P-SH-wave propagation in plates, and constitution of smart technical devices. This can usher gravitation into a new experimental and industrial era.

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Appendix I.

To find all the suitable eigenvectors corresponding to found eigenvalues (52) and (53), it is necessary to treat equations' set (50) anew. It is natural to utilize the first equation in set (50) for determination of the first eigenvector component U^0 as a function of the rest components φ^0, ψ^0, Φ^0, and Ψ^0. So, this dependence reads:

$$U^0 = -m\left(e\varphi^0 + h\psi^0 + g\Phi^0 + f\Psi^0\right)/CA \quad (A1)$$

where $m = 1 + n_3^2$ and

$$\begin{cases} A = m - \left(V_{ph}/V_{t4}\right)^2 = -\left(V_{ph}/V_{t4}\right)^2 \text{ for eigenvalue (52)} \\ A = m - \left(V_{ph}/V_{t4}\right)^2 = -mK_{emgc}^2 \text{ for eigenvalue (53)} \end{cases} \quad (A2)$$

Utilization of definition (A1) for equations' set (50) allows one to exclude the eigenvector component U^0 from the further consideration and to deal with a reduced set of equations. This is the usual mathematical procedure for finding of the unknowns for the set of five equations in five unknowns. It is also useful to state that formulae (A2) are also applicable for the problem of finding of suitable eigenvectors when the wave propagation in piezoelectromagnetics (Zakharenko, 2010; Zakharenko, 2013a,b; Zakharenko, 2015a,b) (i.e. $K_{emgc}^2 \rightarrow K_{em}^2$) is investigated.

I.A1. The first order of equations

So, the new set of four homogeneous equations can be written as follows:

$$\begin{cases} \varepsilon\left(1+mK_e^2/A\right)\varphi^0+\alpha\left(1+mK_\alpha^2/A\right)\psi^0 \\ +\zeta\left(1+mK_\zeta^2/A\right)\Phi^0+\xi\left(1+mK_\xi^2/A\right)\Psi^0=0 \\ \alpha\left(1+mK_\alpha^2/A\right)\varphi^0+\mu\left(1+mK_m^2/A\right)\psi^0 \\ +\beta\left(1+mK_\beta^2/A\right)\Phi^0+\lambda\left(1+mK_\lambda^2/A\right)\Psi^0=0 \\ \zeta\left(1+mK_\zeta^2/A\right)\varphi^0+\beta\left(1+mK_\beta^2/A\right)\psi^0 \\ +\gamma\left(1+mK_g^2/A\right)\Phi^0+\vartheta\left(1+mK_\vartheta^2/A\right)\Psi^0=0 \\ \xi\left(1+mK_\xi^2/A\right)\varphi^0+\lambda\left(1+mK_\lambda^2/A\right)\psi^0 \\ +\vartheta\left(1+mK_\vartheta^2/A\right)\Phi^0+\eta\left(1+mK_f^2/A\right)\Psi^0=0 \end{cases} \quad \text{(A3)}$$

where

$$K_e^2 = e^2/C\varepsilon \quad \text{(A4)}$$

$$K_\alpha^2 = eh/C\alpha \quad \text{(A5)}$$

$$K_\zeta^2 = eg/C\zeta \quad \text{(A6)}$$

$$K_\xi^2 = ef/C\xi \quad \text{(A7)}$$

$$K_m^2 = h^2/C\mu \quad \text{(A8)}$$

$$K_\beta^2 = hg/C\beta \quad \text{(A9)}$$

$$K_\lambda^2 = hf/C\lambda \quad \text{(A10)}$$

$$K_g^2 = g^2/C\gamma \quad \text{(A11)}$$

$$K_\vartheta^2 = gf/C\vartheta \quad \text{(A12)}$$

$$K_f^2 = f^2/C\eta \quad \text{(A13)}$$

Next, from the first equation in set (A3) it is possible to determine the second eigenvector component φ^0 as a function of the components ψ^0, Φ^0, and Ψ^0. It can be composed as follows:

$$\varphi^0 = -\frac{\alpha\left(A+mK_\alpha^2\right)}{\varepsilon\left(A+mK_e^2\right)}\psi^0-\frac{\zeta\left(A+mK_\zeta^2\right)}{\varepsilon\left(A+mK_e^2\right)}\Phi^0-\frac{\xi\left(A+mK_\xi^2\right)}{\varepsilon\left(A+mK_e^2\right)}\Psi^0 \quad \text{(A14)}$$

Definition (A14) for φ^0 can be then utilized in set (A3) to reduce the set of four homogeneous equations in four undetermined. Indeed, it is natural to use definition (A14) for the second, third, and fourth equations in set (A3). As a result, the new reduced set of three homogeneous equations with three unknown components ψ^0, Φ^0, and Ψ^0 read:

$$\begin{cases} \left(\frac{\mu\left(A+mK_m^2\right)}{A}-\frac{\alpha^2\left(A+mK_\alpha^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}\right)\psi^0 \\ +\left(\frac{\beta\left(A+mK_\beta^2\right)}{A}-\frac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Phi^0 \\ +\left(\frac{\lambda\left(A+mK_\lambda^2\right)}{A}-\frac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Psi^0=0 \\ \left(\frac{\beta\left(A+mK_\beta^2\right)}{A}-\frac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\psi^0 \\ +\left(\frac{\gamma\left(A+mK_g^2\right)}{A}-\frac{\zeta^2\left(A+mK_\zeta^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}\right)\Phi^0 \\ +\left(\frac{\vartheta\left(A+mK_\vartheta^2\right)}{A}-\frac{\xi\zeta\left(A+mK_\xi^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Psi^0=0 \\ \left(\frac{\lambda\left(A+mK_\lambda^2\right)}{A}-\frac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\psi^0 \\ +\left(\frac{\vartheta\left(A+mK_\vartheta^2\right)}{A}-\frac{\xi\zeta\left(A+mK_\xi^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Phi^0 \\ +\left(\frac{\eta\left(A+mK_f^2\right)}{A}-\frac{\xi^2\left(A+mK_\xi^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}\right)\Psi^0=0 \end{cases} \quad \text{(A15)}$$

Exploiting the first equation in set (A15), the third eigenvector component ψ^0 represents the following function of the eigenvector components Φ^0 and Ψ^0:

$$\psi^0 = -\frac{\beta\left(A+mK_\beta^2\right)-\frac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{\varepsilon\left(A+mK_e^2\right)}}{\mu\left(A+mK_m^2\right)-\frac{\alpha^2\left(A+mK_\alpha^2\right)^2}{\varepsilon\left(A+mK_e^2\right)}}\Phi^0 -\frac{\lambda\left(A+mK_\lambda^2\right)-\frac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{\varepsilon\left(A+mK_e^2\right)}}{\mu\left(A+mK_m^2\right)-\frac{\alpha^2\left(A+mK_\alpha^2\right)^2}{\varepsilon\left(A+mK_e^2\right)}}\Psi^0 \quad \text{(A16)}$$

Finally, function $\psi^0(\Phi^0, \Psi^0)$ (A16) must be used for substitution in the second and third equations in set (A15). This substitution results in the final two homogeneous equations in two unknowns Φ^0 and Ψ^0 that already can be readily used for definition of both Φ^0 and Ψ^0. These two complicated equations can be composed in the following forms:

$$\begin{cases} 0=\left(\dfrac{\gamma\left(A+mK_g^2\right)}{A}-\dfrac{\zeta^2\left(A+mK_\zeta^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}-\dfrac{\left(\dfrac{\beta\left(A+mK_\beta^2\right)}{A}-\dfrac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)^2}{\dfrac{\mu\left(A+mK_m^2\right)}{A}-\dfrac{\alpha^2\left(A+mK_\alpha^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}}\right)\Phi^0 \\ +\left(\dfrac{\vartheta\left(A+mK_\vartheta^2\right)}{A}-\dfrac{\xi\zeta\left(A+mK_\xi^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Psi^0 \\ -\dfrac{\left(\dfrac{\beta\left(A+mK_\beta^2\right)}{A}-\dfrac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\left(\lambda\left(A+mK_\lambda^2\right)-\dfrac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{\varepsilon\left(A+mK_e^2\right)}\right)}{\mu\left(A+mK_m^2\right)-\dfrac{\alpha^2\left(A+mK_\alpha^2\right)^2}{\varepsilon\left(A+mK_e^2\right)}}\Psi^0 \\ 0=\left(\dfrac{\vartheta\left(A+mK_\vartheta^2\right)}{A}-\dfrac{\xi\zeta\left(A+mK_\xi^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\Phi^0 \\ -\dfrac{\left(\dfrac{\beta\left(A+mK_\beta^2\right)}{A}-\dfrac{\alpha\zeta\left(A+mK_\alpha^2\right)\left(A+mK_\zeta^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)\left(\lambda\left(A+mK_\lambda^2\right)-\dfrac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{\varepsilon\left(A+mK_e^2\right)}\right)}{\mu\left(A+mK_m^2\right)-\dfrac{\alpha^2\left(A+mK_\alpha^2\right)^2}{\varepsilon\left(A+mK_e^2\right)}}\Phi^0 \\ +\left(\dfrac{\eta\left(A+mK_f^2\right)}{A}-\dfrac{\xi^2\left(A+mK_\xi^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}-\dfrac{\left(\dfrac{\lambda\left(A+mK_\lambda^2\right)}{A}-\dfrac{\alpha\xi\left(A+mK_\alpha^2\right)\left(A+mK_\xi^2\right)}{A\varepsilon\left(A+mK_e^2\right)}\right)^2}{\dfrac{\mu\left(A+mK_m^2\right)}{A}-\dfrac{\alpha^2\left(A+mK_\alpha^2\right)^2}{A\varepsilon\left(A+mK_e^2\right)}}\right)\Psi^0 \end{cases} \tag{A17}$$

Equations' set (A17) represents a set of two homogeneous equations in two unknowns Φ^0 and Ψ^0. This pair of equations can be schematically written as follows: $a_1x + by = 0$ and $bx + a_2y = 0$. Therefore, the unknowns x and y can be chosen in two different ways: (1) $x = -b$ and $y = a_1$; (2) $x = a_2$ and $y = -b$. Taking into account this fact it is natural to write down below two different sets (i1) and (ii1) of the eigenvector components for the case of equations (A3).

(i1) The first eigenvectors for case (A3)

The first eigenvectors can be composed with the first equation in set (A17) and definitions (A1), (A14), and (A16). For eigenvalues (52), $m = 0$ and therefore, the corresponding eigenvector components are relatively simple. So, these eigenvector components are

$$\begin{pmatrix} U^{0(1)} \\ \varphi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \varphi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \varphi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(7)} \\ \varphi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix}$$

$$= \begin{pmatrix} U^0 = 0 \\ \varphi^0 = -\dfrac{\alpha}{\varepsilon}\psi^0 - \dfrac{\zeta}{\varepsilon}\Phi^0 - \dfrac{\xi}{\varepsilon}\Psi^0 \\ \psi^0 = -\dfrac{\beta - \dfrac{\alpha\zeta}{\varepsilon}}{\mu - \dfrac{\alpha^2}{\varepsilon}}\Phi^0 - \dfrac{\lambda - \dfrac{\alpha\xi}{\varepsilon}}{\mu - \dfrac{\alpha^2}{\varepsilon}}\Psi^0 \\ \Phi^0 = \vartheta - \dfrac{\xi\zeta}{\varepsilon} - \dfrac{\left(\beta - \dfrac{\alpha\zeta}{\varepsilon}\right)\left(\lambda - \dfrac{\alpha\xi}{\varepsilon}\right)}{\mu - \dfrac{\alpha^2}{\varepsilon}} \\ \Psi^0 = -\gamma + \dfrac{\zeta^2}{\varepsilon} + \dfrac{\left(\beta - \dfrac{\alpha\zeta}{\varepsilon}\right)^2}{\mu - \dfrac{\alpha^2}{\varepsilon}} \end{pmatrix} \tag{A18}$$

However, for eigen value (53) there is a more complicated eigenvector. For this case, the parameter A defined by expression (A2) does not depend on the propagation velocity. Therefore, the utilization of the corresponding parameter A (A2), the first equation in set (A17), and definitions (A1), (A14), (A16) leads to the following complicated eigenvector components:

$$\begin{pmatrix} U^{0(9)} \\ \varphi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^0 = \left(e\varphi^0 + h\psi^0 + g\Phi^0 + f\Psi^0\right)/CK_{emgc}^2 \\ \varphi^0 = -\dfrac{\alpha K_A}{\varepsilon K_E}\psi^0 - \dfrac{\zeta K_Z}{\varepsilon K_E}\Phi^0 - \dfrac{\xi K_S}{\varepsilon K_E}\Psi^0 \\ \psi^0 = -\dfrac{\beta K_B - \dfrac{\alpha\zeta K_A K_Z}{\varepsilon K_E}}{\mu K_M - \dfrac{\alpha^2 K_A{}^2}{\varepsilon K_E}}\Phi^0 - \dfrac{\lambda K_L - \dfrac{\alpha\xi K_A K_S}{\varepsilon K_E}}{\mu K_M - \dfrac{\alpha^2 K_A{}^2}{\varepsilon K_E}}\Psi^0 \\ \Phi^0 = \dfrac{\vartheta K_T}{K_{emgc}^2} - \dfrac{\xi\zeta K_S K_Z}{\varepsilon K_E K_{emgc}^2} - \dfrac{\left(\dfrac{\beta K_B}{K_{emgc}^2} - \dfrac{\alpha\zeta K_A K_Z}{\varepsilon K_E K_{emgc}^2}\right)\left(\lambda K_L - \dfrac{\alpha\xi K_A K_S}{\varepsilon K_E}\right)}{\mu K_M - \dfrac{\alpha^2 K_A{}^2}{\varepsilon K_E}} \\ \Psi^0 = -\dfrac{\gamma K_G}{K_{emgc}^2} + \dfrac{\zeta^2 K_Z{}^2}{\varepsilon K_E K_{emgc}^2} + \dfrac{\left(\dfrac{\beta K_B}{K_{emgc}^2} - \dfrac{\alpha\zeta K_A K_Z}{\varepsilon K_E K_{emgc}^2}\right)^2}{\dfrac{\mu K_M}{K_{emgc}^2} - \dfrac{\alpha^2 K_A{}^2}{\varepsilon K_E K_{emgc}^2}} \end{pmatrix} \tag{A19}$$

where

$$K_E = K_{emgc}^2 - K_e^2 \tag{A20}$$

$$K_M = K_{emgc}^2 - K_m^2 \tag{A21}$$

$$K_G = K_{emgc}^2 - K_g^2 \tag{A22}$$

$$K_F = K_{emgc}^2 - K_f^2 \quad \text{(A23)}$$

$$K_A = K_{emgc}^2 - K_\alpha^2 \quad \text{(A24)}$$

$$K_T = K_{emgc}^2 - K_\vartheta^2 \quad \text{(A25)}$$

$$K_B = K_{emgc}^2 - K_\beta^2 \quad \text{(A26)}$$

$$K_Z = K_{emgc}^2 - K_\zeta^2 \quad \text{(A27)}$$

$$K_S = K_{emgc}^2 - K_\xi^2 \quad \text{(A28)}$$

$$K_L = K_{emgc}^2 - K_\lambda^2 \quad \text{(A29)}$$

(ii1) The second eigenvectors for case (A3)

To obtain the second eigenvectors, it is necessary to use the same equations that were used for the composition of the first eigenvectors, but the first equation in set (A17). Here, the second equation in set (A17) is used instead of the first equation. Therefore, two eigenvectors corresponding to eigenvalues (52) and (53) can be respectively inscribed as follows:

$$\begin{pmatrix} U^{0(1)} \\ \varphi^{0(1)} \\ \psi^{0(1)} \\ \Phi^{0(1)} \\ \Psi^{0(1)} \end{pmatrix} = \begin{pmatrix} U^{0(3)} \\ \varphi^{0(3)} \\ \psi^{0(3)} \\ \Phi^{0(3)} \\ \Psi^{0(3)} \end{pmatrix} = \begin{pmatrix} U^{0(5)} \\ \varphi^{0(5)} \\ \psi^{0(5)} \\ \Phi^{0(5)} \\ \Psi^{0(5)} \end{pmatrix} = \begin{pmatrix} U^{0(7)} \\ \varphi^{0(7)} \\ \psi^{0(7)} \\ \Phi^{0(7)} \\ \Psi^{0(7)} \end{pmatrix}$$

$$= \begin{pmatrix} U^0 = 0 \\ \varphi^0 = -\dfrac{\alpha}{\varepsilon}\psi^0 - \dfrac{\zeta}{\varepsilon}\Phi^0 - \dfrac{\xi}{\varepsilon}\Psi^0 \\ \psi^0 = -\dfrac{\beta - \dfrac{\alpha\zeta}{\varepsilon}}{\mu - \dfrac{\alpha^2}{\varepsilon}}\Phi^0 - \dfrac{\lambda - \dfrac{\alpha\xi}{\varepsilon}}{\mu - \dfrac{\alpha^2}{\varepsilon}}\Psi^0 \\ \Phi^0 = -\eta + \dfrac{\xi^2}{\varepsilon} + \dfrac{\left(\lambda - \dfrac{\alpha\xi}{\varepsilon}\right)^2}{\mu - \dfrac{\alpha^2}{\varepsilon}} \\ \Psi^0 = \vartheta - \dfrac{\xi\zeta}{\varepsilon} - \dfrac{\left(\beta - \dfrac{\alpha\zeta}{\varepsilon}\right)\left(\lambda - \dfrac{\alpha\xi}{\varepsilon}\right)}{\mu - \dfrac{\alpha^2}{\varepsilon}} \end{pmatrix}$$

(A30)

$$\begin{pmatrix} U^{0(9)} \\ \varphi^{0(9)} \\ \psi^{0(9)} \\ \Phi^{0(9)} \\ \Psi^{0(9)} \end{pmatrix} = \begin{pmatrix} U^0 = \left(e\varphi^0 + h\psi^0 + g\Phi^0 + f\Psi^0\right) / CK_{emgc}^2 \\ \varphi^0 = -\dfrac{\alpha K_A}{\varepsilon K_E}\psi^0 - \dfrac{\zeta K_Z}{\varepsilon K_E}\Phi^0 - \dfrac{\xi K_S}{\varepsilon K_E}\Psi^0 \\ \psi^0 = -\dfrac{\beta K_B - \dfrac{\alpha\zeta K_A K_Z}{\varepsilon K_E}}{\mu K_M - \dfrac{\alpha^2 K_A^2}{\varepsilon K_E}}\Phi^0 - \dfrac{\lambda K_L - \dfrac{\alpha\xi K_A K_S}{\varepsilon K_E}}{\mu K_M - \dfrac{\alpha^2 K_A^2}{\varepsilon K_E}}\Psi^0 \\ \Phi^0 = -\dfrac{\eta K_F}{K_{emgc}^2} + \dfrac{\xi^2 K_S^2}{\varepsilon K_E K_{emgc}^2} + \dfrac{\left(\dfrac{\lambda K_L}{K_{emgc}^2} - \dfrac{\alpha\xi K_A K_S}{\varepsilon K_E K_{emgc}^2}\right)^2}{\dfrac{\mu K_M}{K_{emgc}^2} - \dfrac{\alpha^2 K_A^2}{\varepsilon K_E K_{emgc}^2}} \\ \Psi^0 = \dfrac{\vartheta K_T}{K_{emgc}^2} - \dfrac{\xi\zeta K_S K_Z}{\varepsilon K_E K_{emgc}^2} \\ -\dfrac{\left(\dfrac{\beta K_B}{K_{emgc}^2} - \dfrac{\alpha\zeta K_A K_Z}{\varepsilon K_E K_{emgc}^2}\right)\left(\lambda K_L - \dfrac{\alpha\xi K_A K_S}{\varepsilon K_E}\right)}{\mu K_M - \dfrac{\alpha^2 K_A^2}{\varepsilon K_E}} \end{pmatrix}$$

(A31)

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